

Symanzik's Method Applied To The Fractional Quantum Hall Edge States

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Abstract :

In this paper we consider an abelian Chern-Simons theory with plane boundary and we show, following Symankiz's quite general approach, how the known results for edge states in the Laughlin series can be derived in a systematic way by the separability condition. Moreover we show that the conserved boundary currents find a natural and explicit interpretation in terms of the continuity equation and the Tomonaga-Luttinger commutation relation for electronic density is recovered.

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1 Introduction

The condensed matter theoreticians have adopted, quite a few years ago, the three dimensional abelian Chern-Simons gauge model as one, and maybe amongst the most important, ingredient to describe the low energy physics of the Fractional Quantum Hall Effect (FQHE) [1, 2, 3]. As is well known one is dealing with a topological theory that describes, by adding a coupling with an external electromagnetic potential and with a current of gapped quasiparticles, the quantized Hall conductance, the charge and the statistics of these quasiparticles. These results are insensitive to the details of the particular setup of the experimental apparatus and to the quantum fluctuations.

A popular choice is to add a boundary where the gapped quasiparticles become gapless (edge states) [2, 4, 5]. At this point one is faced with the problem of choosing the boundary conditions for the gauge field and of computing the action for the field theory which lives on the boundary.

Many proposals have appeared in the Literature [2, 6, 7]; all of them achieve the goal of describing the wanted properties of the boundary field theory, but the ways to derive them are often based on *ad hoc* assumptions (for instance, the explicit dependence of the results on a particular choice of the gauge fixing term and/or of the boundary conditions).

Here we reconsider the problem with a different approach, which stems from the inclusion of a boundary which meets a precise physical requirement. We would like to emphasize, that we will not find any new result; the novelty is the method itself which fits in a rigorous, perturbative, quantum field theory treatment of the problem. Now the inclusion of a boundary in a Minkowskian or Euclidean field theory is not an easy task if one wishes to preserve locality and power counting, the most basic ingredients of any perturbative quantum field theory model.

Many years ago K. Symanzik [8] proposed a solution: his idea was to add to the classical bulk action a local boundary term which modifies the propagator of the field in such a way that nothing propagates from one side of the boundary to the other. He called this property “separability” and showed that it requires a well identified class of boundary conditions to be realized. Symanzik himself applied the method to compute the Casimir effect for two parallel plates; later on there have been other applications which include the non abelian Chern-Simons model where it is proven that on the boundary there is a set of chiral currents obeying a Kac-Moody algebra with central charge [9]. In this paper we apply Symanzik’s method to the case of an abelian Chern-Simons gauge model with a plane boundary; in order to make the analysis as simple as possible, without losing rigor, we shall adopt the gauge fixing of reference [10]. With this choice one can avoid all technical problems related to the BRS transformations and to the presence of ghost fields. The gauge symmetry, including the boundary contribution which is deduced from the separability condition, is encoded in a local Ward identity;

from it we are able to compute, in a unique way, the propagators on the boundary and therefore we identify the action which describes the theory restricted to the boundary. This action is written in terms of a scalar field which obeys a chirality condition.

Let us emphasize that this result is obtained without any *ad hoc* hypothesis concerning the gauge fixing term and/or the boundary conditions; identical conclusions can be reached by using, for instance, the Landau gauge at the price of some algebraic complications, and the boundary values of the field are given by the separability condition.

The paper is so organized: in Section 2 we describe the bulk model just to fix the notation. The boundary is introduced in Section 3 where we also identify the propagators obeying the separability condition. The corresponding effective bosonic action is given in Section 4. In Section 5 we show that the chiral boundary currents obey a Kac-Moody algebra with central charge. Some conclusive remarks are collected in the final Section, while in the Appendix, the propagator for the two dimensional effective bosonic action is derived.

2 The Model

2.1 The Action

Let us consider the abelian Chern-Simons theory

$$S_{cs} = i \int d^3x \left(\frac{k}{2} \epsilon^{\mu\nu\rho} A_\mu \partial_\nu A_\rho + j^\mu A_\mu \right), \quad (2.1)$$

where k is a coupling constant and $A_\mu(x)$ is a gauge field. The metric of the space is chosen to be flat euclidean, the infinitesimal distance being

$$(ds)^2 = (dx_0)^2 + (dx_1)^2 + (dx_2)^2. \quad (2.2)$$

In (2.1) we introduce a coupling with a generic classical external current j^μ , whose normalization fixes the value of k .

The model defined by (2.1) is consistent with the quantization of the conductance for the Laughlin's sequence [11] of the FQHE, if

$$k = \frac{1}{2\pi\nu} = \frac{2m+1}{2\pi} \quad (2.3)$$

with $m \in \mathbf{N}$ [1]. Here,

$$\nu = \frac{1}{2m+1} \quad (2.4)$$

represents the filling factor of the Quantum Hall fluid for the Laughlin sequence. The relation between k and ν is necessary to describe the right properties of charge and statistic of the quasiparticles that constitute the

Hall fluid.

For the aim of this paper, the presence of the classical current j^μ is irrelevant, and therefore it will be omitted in what follows, just keeping in mind that the coupling constant k cannot be reabsorbed by a redefinition of the gauge fields $A_\mu(x)$. In light-cone coordinates

$$\begin{aligned} u &= x_2 \\ z &= \frac{1}{\sqrt{2}}(x_1 - ivx_0) \\ \bar{z} &= \frac{1}{\sqrt{2}}(x_1 + ivx_0) , \end{aligned} \quad (2.5)$$

where v is a non-relativistic velocity, the Chern-Simons action reads

$$S_{cs} = ik \int dudzd\bar{z} \left(\bar{A} \partial_u A + A_u \partial \bar{A} - A_u \bar{\partial} A \right) , \quad (2.6)$$

where

$$\begin{aligned} A_u &= A_2 \\ A &= \frac{1}{\sqrt{2}}(A_1 + \frac{i}{v}A_0) \\ \bar{A} &= \frac{1}{\sqrt{2}}(A_1 - \frac{i}{v}A_0) . \end{aligned} \quad (2.7)$$

We need now to introduce a gauge fixing term in the action, and we choose an axial gauge condition

$$A_u = 0. \quad (2.8)$$

The ghost fields decouple completely from the gauge field and so can be eliminated. Therefore the gauge fixing term of the action is

$$S_{gf} = i \int dudzd\bar{z} b A_u \quad (2.9)$$

where b is a lagrangian multiplier.

The complete action for the theory that we consider is therefore

$$S = S_{cs} + S_{gf}. \quad (2.10)$$

2.2 Symmetries And Constraints

To each object appearing in the action S , a canonical mass dimension and an “helicity” quantum number is assigned, as shown in Table 1 :

Table 1 : Quantum Numbers

	A_u	A	\bar{A}	b	∂_u	∂	$\bar{\partial}$	u	z	\bar{z}
dim	1	1	1	2	1	1	1	-1	-1	-1
hel	0	1	-1	0	0	1	-1	0	-1	1

The action S is the most general one which obeys the following constraints :

1. S is a dimensionless, local, integrated functional of a polynomial lagrangian density, with helicity zero.
2. S is invariant under the local, infinitesimal gauge transformation

$$\begin{aligned}\delta A_\mu &= \partial_\mu \theta \\ \delta b &= 0\end{aligned}\tag{2.11}$$

where θ is the local gauge parameter.

3. S is invariant under the discrete symmetry involving at the same time coordinates and fields

$$\begin{aligned}z &\leftrightarrow \bar{z} \\ u &\rightarrow -u \\ A &\leftrightarrow \bar{A} \\ A_u &\rightarrow -A_u \\ b &\rightarrow -b\end{aligned}\tag{2.12}$$

2.3 The Generating Functional $Z[J]$

Starting from the action S , we can define, in three dimensional euclidean spacetime, the generating functional of the Green functions

$$Z[J_\chi] = \int \mathcal{D}\chi \exp \left[- \left(S + \int du dz d\bar{z} \sum_\chi J_\chi \chi \right) \right]\tag{2.13}$$

with quantum sources for the gauge fields :

$$\langle \chi(X) \rangle = \left. \frac{\delta Z}{\delta J_\chi(X)} \right|_{J=0}\tag{2.14}$$

where $J_\chi = J_u, \bar{J}, J, J_b$, $\chi = A_u, A, \bar{A}, b$, and $(X) \equiv (u, z, \bar{z})$.

A generic N -point Green function is defined by

$$\langle \chi_1(X_1) \cdots \chi_N(X_N) \rangle = \left. \frac{\delta^N Z}{\delta J_{\chi_1}(X_1) \cdots \delta J_{\chi_N}(X_N)} \right|_{J=0}.\tag{2.15}$$

From the definition of $Z[J]$, we deduce the canonical dimensions and helicities of the source fields, displayed in Table 2 :

Table 2 : Sources

	J_u	J	\bar{J}	J_b
dim	2	2	2	1
hel	0	1	-1	0

We can now derive from (2.13) the equations of motion

$$\begin{aligned}
ik \left(\bar{\partial} A_u - \partial_u \bar{A} \right) + \bar{J} &= 0 \\
ik \left(\partial_u A - \bar{\partial} A_u \right) + J &= 0 \\
ik \left(\partial \bar{A} - \bar{\partial} A + \frac{1}{k} b \right) + J_u &= 0 \\
iA_u + J_b &= 0
\end{aligned} \tag{2.16}$$

which lead to the local Ward identity

$$\partial \bar{J} + \bar{\partial} J + \partial_u J_u + i \partial_u b = 0 . \tag{2.17}$$

Notice that, it is because of the axial gauge condition (2.8) that the gauge symmetry of the model is described by a local Ward identity.

3 Introduction Of A Boundary

Let us now introduce as a boundary the plane

$$u = 0 . \tag{3.1}$$

Following [8, 9, 10], the presence of the boundary is correctly taken into account if the following two constraints are satisfied :

1. Separability: every Green function (2.15), in particular the propagators, must vanish when the points X_n don't lie in the same half-space delimited by the boundary (3.1).
2. Locality: if all the points X_n lie in the same half-space, and none of them is on the boundary, the correlators are the same as in the original theory without boundary.

The two conditions are satisfied by a generating functional of the form

$$Z = Z_+ + Z_- , \tag{3.2}$$

where the indices $+$ and $-$ refer to the half-spaces $u > 0$ and $u < 0$, respectively.

For what concerns the propagators, the above constraints are satisfied if

$$\Delta_{\chi_1 \chi_2}(X_1, X_2) = \langle \chi_1(X_1) \chi_2(X_2) \rangle = \theta_+ \Delta_+(X_1, X_2) + \theta_- \Delta_-(X_1, X_2) \tag{3.3}$$

where

$$\theta_{\pm} \equiv \theta(\pm u_1) \theta(\pm u_2) \tag{3.4}$$

is equal to 1 if u_1 and u_2 are both in the same half-space, and equal to zero otherwise.

The elements of the matrix $\Delta_{\chi_1\chi_2}(X_1, X_2)$ have been determined in [10], and the aim of this paper is to find out which quantum fields can describe the physics on the two-sided two dimensional (2D) boundary, and by means of which 2D action. To reach this goal, we derive the correlation functions on each side of the boundary, starting from the 2D Ward identity expressing the gauge invariance on the boundary, and after that we will look for an effective action which reproduces these correlation functions.

We need to know how the equations of motion (2.16) and the Ward identity (2.17) are modified by the presence of the boundary.

Taking into account power counting and helicity constraints, the presence of the boundary modifies the equations of motion (2.16) as follows [10] :

$$\begin{aligned} ik(\bar{\partial}A_u - \partial_u\bar{A}) + \bar{J} &= ik\delta(u)\bar{A}_- \\ ik(\partial_u A - \bar{\partial}A_u) + J &= ik\delta(u)A_+ \\ ik\left(\partial\bar{A} - \bar{\partial}A + \frac{1}{k}b\right) + J_u &= 0 \\ iA_u + J_b &= 0 , \end{aligned} \quad (3.5)$$

where

$$\bar{A}_{\pm}(Z) \equiv \lim_{u \rightarrow 0^{\pm}} \bar{A}(X) \quad A_{\pm}(Z) \equiv \lim_{u \rightarrow 0^{\pm}} A(X) \quad (3.6)$$

(with $(Z) \equiv (z, \bar{z})$), are the Chern-Simons fields on the boundary, and, as it has been shown in [9, 10], the following Dirichlet boundary conditions must hold

$$\bar{A}_+(Z) = A_-(Z) = 0 . \quad (3.7)$$

Correspondingly, the local Ward identity (2.17) acquires a boundary breaking :

$$\partial\bar{J} + \bar{\partial}J + \partial_u J_u + i\partial_u b = ik\delta(u)(\bar{\partial}A_+ + \partial\bar{A}_-) , \quad (3.8)$$

which, once integrated, gives the 2D Ward identity describing the gauge symmetry on the boundary

$$-\frac{i}{k} \int_{-\infty}^{+\infty} du [\partial\bar{J}(X) + \bar{\partial}J(X)] = \bar{\partial}A_+(Z) + \partial\bar{A}_-(Z) . \quad (3.9)$$

We concentrate ourselves on the side $u \rightarrow 0^+$, the expressions for the opposite side being derived by means of the inversion (2.12).

We begin by the correlator

$$\langle A_+(Z)A_+(Z') \rangle = \lim_{u,u' \rightarrow 0^+} \frac{\delta^2 Z}{\delta\bar{J}(X)\delta\bar{J}(X')} \Big|_{J=0} . \quad (3.10)$$

Applying to both sides of (3.9) the functional operator

$$\lim_{u \rightarrow 0^+} \frac{\delta}{\delta\bar{J}(X)} , \quad (3.11)$$

we obtain the differential equation between correlators

$$-\frac{i}{k}\partial\delta^2(Z-Z')=\bar{\partial}\langle A_+(Z)A_+(Z')\rangle+\partial\langle\bar{A}_-(Z)A_+(Z')\rangle. \quad (3.12)$$

Remembering the separability condition according to which correlation functions between points belonging to different half-spaces vanish, using the helicity and power counting constraints, and, finally, exploiting the relation

$$\delta^2(Z-Z')=\frac{1}{2\pi i}\bar{\partial}\frac{1}{z-z'} \quad (3.13)$$

between tempered distributions, one has

$$\langle A_+(Z)A_+(Z')\rangle=\frac{1}{2\pi k}\frac{1}{(z-z')^2}. \quad (3.14)$$

Notice that the above correlator turns out to be chiral. Proceeding in an analogous way, from (3.9) we obtain also

$$-\frac{i}{k}\bar{\partial}\delta^2(Z-Z')=\bar{\partial}\langle A_+(Z)\bar{A}_+(Z')\rangle+\partial\langle\bar{A}_-(Z)\bar{A}_+(Z')\rangle, \quad (3.15)$$

which gives

$$\langle A_+(Z)\bar{A}_+(Z')\rangle=-\frac{i}{k}\delta^2(Z-Z'). \quad (3.16)$$

For what concerns the propagator $\langle\bar{A}_+(Z)\bar{A}_+(Z')\rangle$, it is set to zero, since it is not generated by the Ward identity (3.9) :

$$\langle\bar{A}_+(Z)\bar{A}_+(Z')\rangle=0. \quad (3.17)$$

The following boundary correlator matrix summarizes our results :

$$\begin{pmatrix} \langle A_+(Z)A_+(Z')\rangle & \langle A_+(Z)\bar{A}_+(Z')\rangle \\ \langle\bar{A}_+(Z)A_+(Z')\rangle & \langle\bar{A}_+(Z)\bar{A}_+(Z')\rangle \end{pmatrix}=\begin{pmatrix} \frac{1}{2\pi k}\frac{1}{(z-z')^2} & -\frac{i}{k}\delta^2(Z-Z') \\ -\frac{i}{k}\delta^2(Z-Z') & 0 \end{pmatrix} \quad (3.18)$$

We are left now with the task of finding an effective 2D theory on the boundary, which reproduces the correlation functions (3.18).

4 Effective Bosonic Action

Let us consider now the 2D action S_B , living on the external side of the boundary $u=0^+$:

$$S_B^{(+)}=i\frac{k}{2}\int dzd\bar{z}\left[\partial\varphi_+\bar{\partial}\varphi_+(\bar{\partial}\varphi_+)^2\right], \quad (4.1)$$

where $\varphi_+(Z)$ is a bosonic field whose propagator is (see Appendix A) :

$$G(z - z') = \langle \varphi_+(Z) \varphi_+(Z') \rangle = \frac{1}{2\pi k} \ln \frac{(z - z')}{\mu}. \quad (4.2)$$

From (4.2), the following correlators are easily derived :

$$\langle \partial \varphi_+(Z) \partial \varphi_+(Z') \rangle = \frac{1}{2\pi k} \frac{1}{(z - z')^2} \quad (4.3)$$

$$\langle \partial \varphi_+(Z) \bar{\partial} \varphi_+(Z') \rangle = -\frac{i}{k} \delta^2(Z - Z') \quad (4.4)$$

$$\langle \bar{\partial} \varphi_+(Z) \bar{\partial} \varphi_+(Z') \rangle = 0, \quad (4.5)$$

which coincide with those appearing in (3.18). We are thus led to identify the components of the gauge fields on the boundary $u = 0^+$ with the derivative of a bosonic field, through the relations

$$A_+(Z) \leftrightarrow \partial \varphi_+(Z) \quad (4.6)$$

$$\bar{A}_+(Z) \leftrightarrow \bar{\partial} \varphi_+(Z). \quad (4.7)$$

Coming back to the original, euclidean, coordinates (x_0, x_1, x_2) , the chiral action (4.1) reads

$$S_B^{(+)} = -\frac{k}{2} \int dx_1 dx_0 [(\partial_1 \varphi_+) (v \partial_1 \varphi_+ - i \partial_0 \varphi_+)] , \quad (4.8)$$

whose equation of motion is

$$(v \partial_1 - i \partial_0) \partial_1 \varphi_+ = 0 \quad (4.9)$$

which implies the chirality of $\partial_1 \varphi_+$. In this case the direction of the propagation for the field is regressive.

We stress that the equation of motion (4.9) alone does not imply chirality also for the undifferentiated bosonic field $\varphi_+(Z)$. In order to be able to claim that the bosonic field is indeed a chiral function of the coordinate z only, some additional boundary conditions must be invoked [5, 12]. For what concerns the internal side of the boundary, it is straightforward to obtain another bosonic action, related to (4.1) by the inversion (2.12), depending on a bosonic field $\varphi_-(Z)$, whose equation of motion implies a progressive propagation for $\partial_1 \varphi_-$.

From the expression (4.8), we have the hamiltonian

$$H = v \frac{k}{2} \int dx_1 (\partial_1 \varphi_+)^2 , \quad (4.10)$$

which, coincides with the one proposed by Wen [1] to describe the edge states in the FQHE for the Laughlin's sequence by imposing

$$k = \frac{1}{2\pi\nu} \quad (4.11)$$

as in equation (2.3).

Therefore, an abelian Chern-Simons gauge field theory with boundary admits, on the opposite sides of the boundary, a description in term of a chiral bosonic field whose propagation is regressive for the external side of the boundary and progressive for the internal side.

The importance of our result is that we derived it only by using the Symanzik's separability condition and by the identification of the correlators of the gauge field with the second derivative of the propagator of a scalar field living on the boundary. We don't impose any other additional condition on the boundary.

5 Conserved Boundary Currents

Let us consider the following operators defined on the external side of the boundary $u \rightarrow 0^+$

$$K_+(Z) \equiv \frac{1}{2\pi} \lim_{u \rightarrow 0^+} A(X) \quad (5.1)$$

$$\bar{K}_+(Z) \equiv \frac{1}{2\pi} \lim_{u \rightarrow 0^+} \bar{A}(X) , \quad (5.2)$$

while the analogous expressions $K_-(Z)$ and $\bar{K}_-(Z)$, as usual, can be derived by the inversion (2.12).

From (3.7) and (3.9), we have [9, 10]

$$\bar{\partial} K_+(Z) = 0 \quad (5.3)$$

$$\bar{K}_+(Z) = 0 . \quad (5.4)$$

The operators (5.1) and (5.2) satisfy the following commutation relation

$$[K_+(z), K_+(z')] = -\frac{i}{(2\pi)^2 k} \partial \delta(z - z') , \quad (5.5)$$

as it can be immediately derived from the Ward identity (3.9). This commutation relation is the remnant of the Kac-Moody algebra formed by the nonabelian counterpart of the chiral conserved currents $K_+(z)$ [9, 10]. Under this respect, the coefficient $-\frac{i}{(2\pi)^2 k}$ can be seen as the central charge of the Kac-Moody algebra.

Moreover, (5.3) and (5.4) obviously yield

$$\bar{\partial} K_+ + \partial \bar{K}_+ = 0 , \quad (5.6)$$

which is easily interpreted as a conservation relation. In fact, coming back to the euclidean coordinates, we can write K_+ and \bar{K}_+ in terms of new

quantities ρ_+ and J_+ , respectively time and space components of a vector in the euclidean spacetime, in analogy with (2.7) :

$$K_+ \equiv \frac{1}{\sqrt{2}v} (J_+ + iv\rho_+) \quad (5.7)$$

$$\bar{K}_+ \equiv \frac{1}{\sqrt{2}v} (J_+ - iv\rho_+) . \quad (5.8)$$

In terms of ρ_+ and J_+ , the relation (5.6) becomes

$$\partial_1 J_+ + \partial_0 \rho_+ = 0 , \quad (5.9)$$

which is the familiar continuity equation involving a density ρ_+ and a current J_+ .

From (5.3) and (5.4), we also get the chirality of ρ_+

$$v\partial_1 \rho_+ - i\partial_0 \rho_+ = 0 \quad (5.10)$$

and the identification of ρ_+ and J_+ :

$$J_+ = iv\rho_+ . \quad (5.11)$$

On the other hand, the commutation relation (5.5), in terms of the density $\rho_+(z)$ is

$$[\rho_+(z), \rho_+(z')] = i\frac{\nu}{4\pi} \partial\delta(z - z') , \quad (5.12)$$

where we introduced the filling factor ν by means of (4.11). The commutation relation (5.12), derived here in the framework of a gauge field theory with boundary, quite remarkably turns out to coincide with the relation peculiar of the Tomonaga-Luttinger theory [13, 14, 15] for a 1+1 dimensional liquid of interacting electrons.

Finally, comparing (5.10) with the equation of motion (4.9), we can make the identification between real hermitian operators

$$\rho_+ = -\frac{i}{2\pi} \partial_1 \varphi_+ . \quad (5.13)$$

Coming back to the Minkowskian spacetime, the electronic density reads

$$\rho_+^M = -\frac{1}{2\pi} \partial_1 \varphi_+ . \quad (5.14)$$

In the usual model of the edge states of the FQHE [1], ρ_+^M corresponds to the electron density on the edge the Hall bar and the above relation allows to connect this physical quantity to the chiral bosonic field that propagate along the edge.

6 Conclusions

In this paper we have shown how the well known results concerning the edge states in the FQHE can be deduced by means of Symanzik's general approach to Quantum Field Theories with boundary. In particular, we derived the bosonic chiral hamiltonian written in terms of an electronic density, and the Luttinger-Tomonaga commutation relations for an electronic liquid in 1+1 dimensions.

The main point we tried to clarify, is that there is a method which respects all the basic assumptions of perturbative quantum field theory, by which all the rest can be deduced. This not only reinforces the validity of the known results, but also gives us hope of going further and apply the same method to describe more complex sequences of the FQHE as the Jain sequence [16] or non abelian states.

Work is in progress in these directions [17].

A Bosonic Chiral Propagator

In this appendix we want to derive the expression (4.2) for the correlator of the bosonic field.

From the action (4.1) we obtain the differential equation for the Green function $G(Z)$

$$-k(-i\partial_0 + v\partial_1)\partial_1 G(Z) = \frac{v}{i}\delta^2(Z) . \quad (\text{A.1})$$

Introducing the function

$$f(Z) = \partial_1 G(Z) \quad (\text{A.2})$$

that satisfies the relation

$$\bar{\partial}f(Z) = \frac{i}{\sqrt{2}k}\delta^2(Z) , \quad (\text{A.3})$$

we find the solution of (A.3)

$$f(z) = \frac{1}{2\sqrt{2}\pi k} \frac{1}{z} . \quad (\text{A.4})$$

Notice that f is a chiral function in agreement with the result in Section 5. Finally we can integrate over x_1 to obtain

$$G(z) = \frac{1}{2\pi k} \ln\left(\frac{z}{\mu}\right) \quad (\text{A.5})$$

where we introduce the arbitrary scale μ . In this work we identify the derivatives of $G(z)$ with the correlators of the Chern-Simons abelian field on the boundary, therefore this parameter doesn't appear in these physical observables.

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